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THE FRACTAL STATISTICS OF LIQUID SLUG LENGTHS

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Abstract—A rescaled range (R/S) analysis is presented of gas–liquid and gas–liquid–liquid slug length data from two extremely different pipeline systems with i.d. = 32 and 189 mm and length-to-diameter ratios of about 800 and 2000, respectively. The results indicate that slug lengths in horizontal pipe flow obey fractal statistics. The R/S analysis of 30–250 slugs for each experiment shows that the length spectrum satisfies Hurst's law with a Hurst exponent in the range 0.53–0.76, linearly dependent on the total superficial velocity. Conversely, if the Hurst exponent and average slug length with standard deviation are given, slug length spectra may be obtained in good agreement with experimental data.

Key Words: fractals, rescaled range analysis, Hurst exponent, successive random additions

1. INTRODUCTION

Gas–liquid slug flow is a frequently occurring regime in horizontal or inclined pipes, visualized in figure 1. This flow type is characterized by sequences of a large gas bubble propagating over a thin film being scooped up by a liquid slug occupying the entire pipe cross section (e.g. Dukler & Hubbard 1975; Bendiksen 1984). Slug flow is the result of hydrodynamic instabilities of the liquid film in stratified or annular flow because of catastrophic growth or coalescence of non-linear gravitational waves above some critical gas flow rate, as described by, for example, Lin & Hanratty (1986).

In practical applications, frequently a second immiscible liquid is present, e.g. in gas–water–oil flow. In this case, the upper oil film primarily causes the instability although internal waves are also present at the oil–water interface. The flow in the liquid slug, which may be aerated in this case, is very complex, as shown by Nuland *et al.* (1989).

Gas–liquid or gas–liquid–liquid slug flows have long been known to exhibit statistical features. For example, liquid slug length distributions have been the subject of a number of studies, including Scott *et al.* (1987) and Brill *et al.* (1981).

It has been thought that slug flow is a purely transient flow regime. The trailing large gas bubbles propagating through adjacent slugs of different length experience different local radial liquid velocity profiles, causing slightly different bubble velocities, which leads to either slug coalescence or decay. Actually, in horizontal pipes it has been argued that slug flow never attains a steady-state behavior, regardless of pipe length, as described by Taitel (1987) and Bendiksen *et al.* (1988). In inclined flows the situation is different, as the gravitational component along the flow direction has a stabilizing effect.

In the present study we are concerned with horizontal flows or with developing inclined flows, as in these cases the upstream flow conditions of adjacent slugs are expected to influence the development of any given slug. It may be speculated that in long inclined pipelines the gravitational component along the flow direction counteracts the past time history of each individual slug. The extent to which the flow is fully developed has been investigated by analyzing data at different flow rates. This has been studied by comparing data from very different horizontal two- and three-phase pipeline systems with lengths of 400 and 26 m, respectively.

2. METHOD

The choice of statistical method is essential for correct physical interpretation of the experimental results. Fractal statistics is a valuable tool for analyzing stochastic fluctuations of time records or

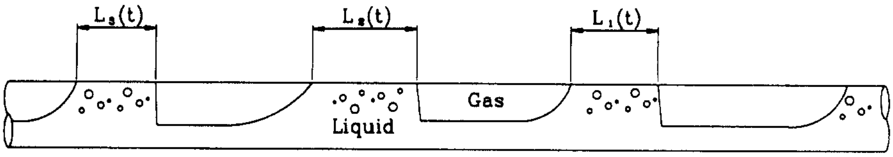

Figure 1. Typical slug flow.

a series of observations. Originally used by Mandelbrot (1983) for studying fluctuations of commodity prices and transmission noise in telephone lines, it has been successfully applied to analyzing records in time of such phenomena as rainfall, discharge of rivers, temperatures and wave heights (see Feder 1988).

One way to describe the fractal statistics is through the so-called rescaled range (R/S) analysis, first used by Hurst (1951) in studying the long-term storage capacity of reservoirs. The rescaled range is defined as $R(s)/S$, where $R(s)$ is the cumulated range of a process and S is the corresponding sample standard deviation. In this work the following procedure was used: given s adjacent slugs with length $L(t)$ each, the average slug length $\bar{L}_s$ for these s slugs may be expressed as

$$\bar{L}_s = \frac{1}{s} \sum_{t=1}^{s} L(t), \qquad [1]$$

where $L(t)$ is the length of the slug at time t.

The accumulated weighted slug length is defined by

$$X(t, s) = \sum_{u=1}^{t} [L(u) - \bar{L}_s]. \qquad [2]$$

The range is the difference between the maximum and the minimum accumulated weighted slug length in the interval,

$$R(s) = \max_{1 \leqslant t \leqslant s} X(t, s) - \min_{1 \leqslant t \leqslant s} X(t, s), \qquad [3]$$

and the standard deviation is defined as

$$S = \sqrt{\frac{1}{s^2} \sum_{t=1}^{s} [L(t) - \bar{L}_s]^2}. \qquad [4]$$

It has been empirically observed that the rescaled range for many records is well-described by the scaling relationship

$$\frac{R}{S} \sim s^H \qquad [5]$$

in our case for s slugs.

The value of the exponent H is in the range $0 < H < 1$, and is directly related to the Hurst exponent defined for fractional Brownian motion (fBm) and fractional Brownian noise.

It has been shown by Mandlebrot (1983) that the value of $H = 1/2$ corresponds to a random process in which case there is no correlation of past and future increments. The situation is, however, quite different for $H \neq 1/2$. For $H > 1/2$ an increasing (decreasing) trend in the past implies an increasing (decreasing) trend in the future, and one speaks of "persistence". For $H < 1/2$ one speaks of "antipersistence", whereby an increasing (decreasing) trend in the past implies a decreasing (increasing) trend in the future. It has been observed that most long-run properties of naturally occurring phenomena exhibit persistence (see Hurst 1951; Feder 1988), with $1/2 < H < 1$.

In the present study the slug length R/S analysis was performed by a specially coded computer program, verified against the data of Hurst (1951). First simulations of a random Gaussian distribution were performed. A random number generator was used to generate 10 random numbers between 0.0 and 1.00. These numbers were then sorted into two categories of numbers above and below 0.5, and the accumulated entries in each category were counted. The counts in each category were then subtracted from each other, and the results recorded. The entire procedure was repeated 50,000 times. The resulting R/S curve turned out as $R/S = [(1.15 \pm 0.07)s]^{0.50 \pm 0.08}$, for all $s > 25$, which is close to the expected $R/S = \sqrt{(\pi s)/2}$ (see Feder 1988). The R/S analysis was also tested on data reported by Hurst (1951) describing the influx and discharge of water in Lake Albert. For this case we obtained $R/S = [(0.76 \pm 0.06)s]^{0.75 \pm 0.03}$, in good agreement with the data of Hurst *et al.* (1965) on river discharges ($H = 0.72 \pm 0.091$).

The solution of the "inverse" problem; i.e. the generation of slug length distributions, is considerably more difficult. Our approach is to create a discrete slug length spectrum based on fBm. For a given flow condition it is then assumed that the Hurst exponent may be expressed in terms of two-phase flow parameters, and that the average liquid slug length ($\bar{L}_s$) with standard deviation (σ) is known.

Ordinary Brownian motion in one dimension is characterized by the increment, or the displacement (ξ_i), in particle position after i steps (X_i), being given by a Gaussian distribution ($p(s)$)with zero mean and unit variance:

$$\xi_i = s_i p(s_i)\,\mathrm{d}s_i$$

$$p(s) = \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2}s^2\right) \qquad [6]$$

The position of the particle at time t may then be expressed as an integral of white noise:

$$X(t) = \int_{-\infty}^{t} sp(s)\,\mathrm{d}s = \int_{-\infty}^{t} W(s)\,\mathrm{d}s. \qquad [7]$$

The average of the increments is then zero,

$$\langle \xi \rangle = \int_{-\infty}^{\infty} s \exp\left(-\tfrac{1}{2}s^2\right) \mathrm{d}s = 0, \qquad [8]$$

and the normalized variance $V(t - t_0)$ may be shown to satisfy

$$V(t - t_0) = \langle [X(t) - X(t_0)^2] \rangle = |t - t_0| \langle X^2(t_0) \rangle = |t - t_0|^{2H} \sigma_0^2, \qquad [9]$$

where $\sigma_0^2 = \langle X^2(t_0) \rangle$ is the initial variance of increments in position, and $H = 1/2$.

The concept of fBm was introduced by Mandelbrot & Van Ness (1968) as a generalization of the random function $X(t)$ by extending the exponent H to the entire interval $0 < H < 1$. It can be shown (e.g. Feder 1988) that for $H \neq 1/2$ there is a correlation of future and past increments, which is given by

$$C(t) = 2^{2H-1} - 1. \qquad [10]$$

For $H > 1/2$, $C(t) > 0$, and the fBm exhibits persistence, i.e. an increasing trend in the past implies an increasing trend in the future. Furthermore, it has been shown by Mandelbrot & Wallis (1969) that the exponent H in [10] is identical to the Hurst exponent.

This is the basis for our attempt to describe slug length distributions in terms of fBm.

To generate fBm in one dimension a method known as successive random additions is applied, as described by Voss (1985) and Saupe (1988). The variance of the increments is then given by [9] as

$$V(t) = \langle [X(t) - X(0)]^2 \rangle = |t|^{2H} \sigma_0^2, \qquad [11]$$

where the Hurst exponent H determines the shape of the fBm. Consider the sequence of positions $X(t_1), X(t_2), \ldots, X(t_n)$ at times $t_1, \ldots, t_n$. We start the first generation with $n = 3$ at $t_i = 0, 1/2, 1$ and give the positions $X(t_1), X(t_2), X(t_3)$ random additions from a log-normal distribution with zero mean and unit variances as explained below. The use of a log-normal, instead of a Gaussian distribution, results in the slug lengths also obeying a log-normal distribution, in accordance with experimental evidence (e.g. Brill *et al.* 1981). Next, the midpoints of the time intervals become additional times $t_1, \ldots, t_5$ in the next generation, with new positions obtained by random additions $X(t_1), \ldots, X(t_5)$, as indicated in figure 2. This procedure is repeated n times where the positions are obtained by the interpolation and random addition process, see Voss (1985) for further details.

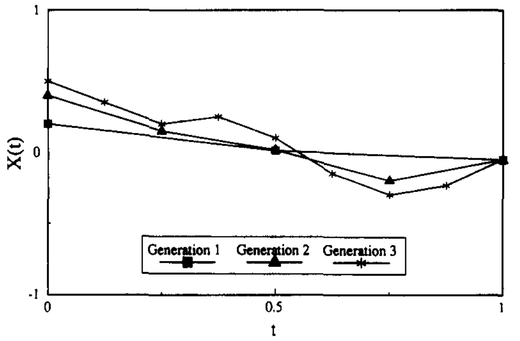

Figure 2. The process of successive random additions.

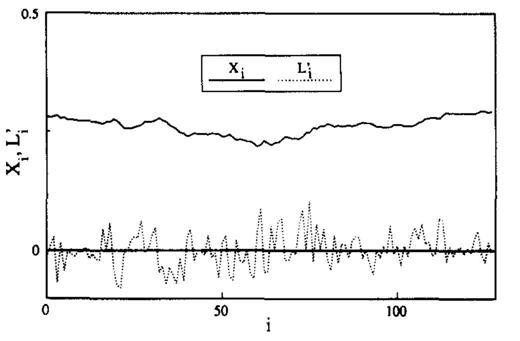

Figure 3. The solid curve (X_i) represents an fBm with $H = 0.7$. The dotted curve (L'_i) represents the difference of the fBm.

The standard deviation of the displacements Δ_i for each iteration i, are given as (Saupe 1988):

$$\Delta_i = \sigma_0 \left(\frac{1}{2}\right)^{H \cdot i} \sqrt{1 - 2^{2H-2}}\, \mathfrak{R}_i, \qquad [12]$$

where σ_0 is the initial standard deviation of the displacements, here $\sigma_0 = 1.0$, and $\mathfrak{R}_i$ are log-normal distributed random numbers.

To generate series of log-normal distributed random numbers, we first pick m different random numbers and calculate their average:

$$\mathfrak{R}'_i = \frac{\sum_{j=1}^{m} \mathfrak{R}_j}{m}, \qquad [13]$$

where m is an integer ($m \geqslant 2$). In our calculations we used $m = 3$. This procedure is repeated until a sufficient amount of random numbers has been generated, and transformed to a sequence of log-normal random numbers by

$$\mathfrak{R}_i = \exp(\mathfrak{R}'_i). \qquad [14]$$

Introducing these random numbers into relation [12] results in a curve describing a fBm. It follows from [7] and [11] using a series expansion that

$$X_i = \sum_{j=1}^{i} L'(t_j), \qquad [15]$$

where $X_i = X(t_i)$ are points on the function describing the fBm. The length L'_i of each slug may then be generated by taking the difference:

$$L'_i = X_{i+1} - X_i. \qquad [16]$$

An example with $H = 0.7$ is shown in figure 3 of fBm (X_i), and the resulting "noise" (L'_i). To obtain the final slug length distribution, it is normalized to the standard deviation of the slug lengths (σ) by multiplying with a constant a, and to the correct average slug length ($\bar{L}_s$) by adding a constant b:

$$L_i = \mathrm{a}L'_i + \mathrm{b}. \qquad [17]$$

Figure 4 shows a plot of slug lengths (L_i), which have $\sigma = 1.5$ and $\bar{L}_s = 5.0$ m. This plot is a result of multiplying the signal in figure 3 by a = 417, and adding b = 4.69.

Figure 5 shows a plot of simulated liquid slug lengths as a function of the slug number for various values of the Hurst exponent; $\sigma = 1.5$ and $\bar{L}_s = 5.0$ m. The sequence of random numbers is the same for each graph. As can be seen from this figure, adjacent slugs for $H = 0.8$ show a higher degree of persistence than for $H = 0.5$ and $H = 0{,}7$. Only 31 out of a total of 128 simulated slugs are shown in the figure.

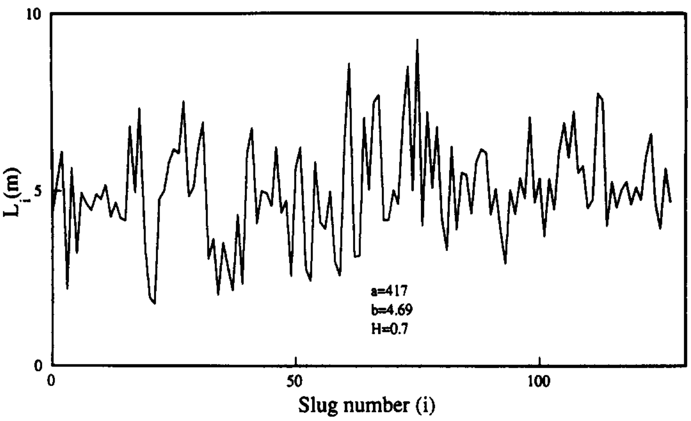

Figure 4. The L_i' curve from figure 3 normalized to $\sigma = 1.5$, with $\bar{L}_s = 5.0$ m. This is achieved by multiplying by a = 417 and adding b = 4.69, as shown in [17].

As a verification, we also performed an R/S analysis by computing the Hurst exponent in [1]–[5] from the data generated by [12]–[17]. A comparison of the obtained Hurst exponent with the corresponding ones for generating the fBm is shown in figure 6, and the agreement is satisfactory.

Finally, figure 7 shows a comparison of a predicted length distribution of 256 generated slugs with an experiment of 243 slugs from the IFE loop (with $H = 0.63$, $\bar{L}_s = 0.87$ m, $\sigma = 0.23$ and $U_m = 7.58$ m/s).

3. EXPERIMENTAL SETUP

Two-phase slug length distributions in time were studied using rescaled range (R/S) analysis, as described above. The R/S analysis of two-phase slug length distributions was based on large-scale, high-pressure data from the SINTEF Two-phase Flow Lab. (figure 8), and data from a small-scale atmospheric air–water–oil loop at IFE (figure 9). The experiments with the SINTEF loop are described by Fuchs & Linga (1984) and by Skarsvåg (1984).

The SINTEF loop has a total horizontal length of 400 m, with i.d. = 189 mm. This horizontal section is connected to a 52 m vertical riser. The reported experiments were run with lube oil and nitrogen at 45 bar. The observed flow regime in the horizontal pipe for the experiments used in this work was always slug flow.

The slug data were recorded at a measuring station positioned 310.5 m downstream from the mixing point. This measuring station consisted of two γ-densitometers which were placed 4 m apart. The recordings were executed with a dwell time of 10 ms. The slug length was obtained by

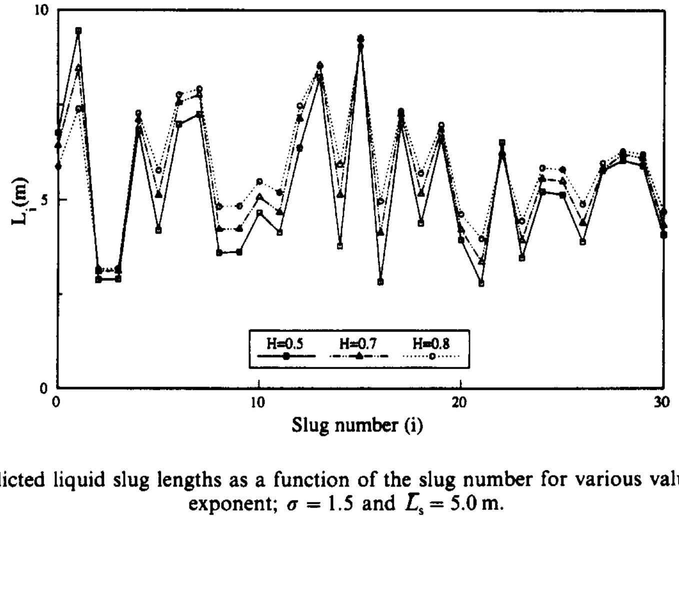

Figure 5. Predicted liquid slug lengths as a function of the slug number for various values of the Hurst exponent; $\sigma = 1.5$ and $\bar{L}_s = 5.0$ m.

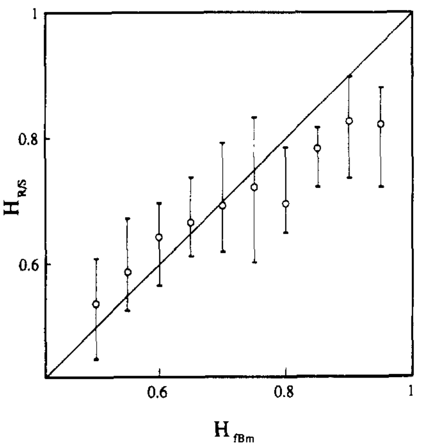

Figure 6. The Hurst exponent from R/S analysis ($H_{R/S}$) vs the corresponding Hurst exponents for fBm (H_{fBm}). For each value of the Hurst exponent, series of 256 slugs were generated 8 times.

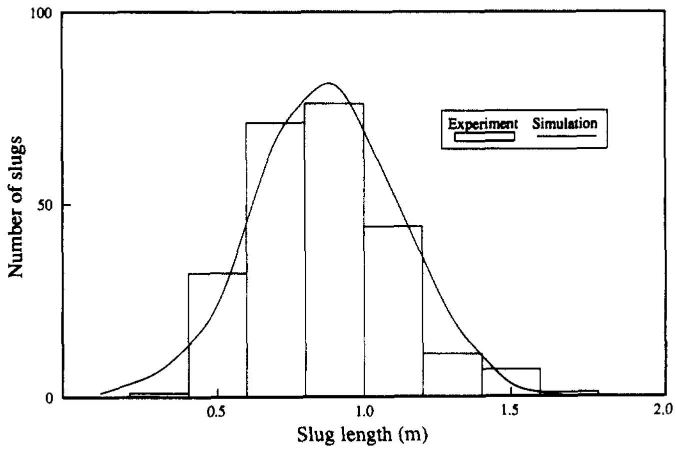

Figure 7. Predicted slug length distribution and data from an experiment from the IFE loop ($U_m = 7.58$ m/s, $H = 0.63$, $\sigma = 0.23$ and $\bar{L}_s = 0.87$ m).

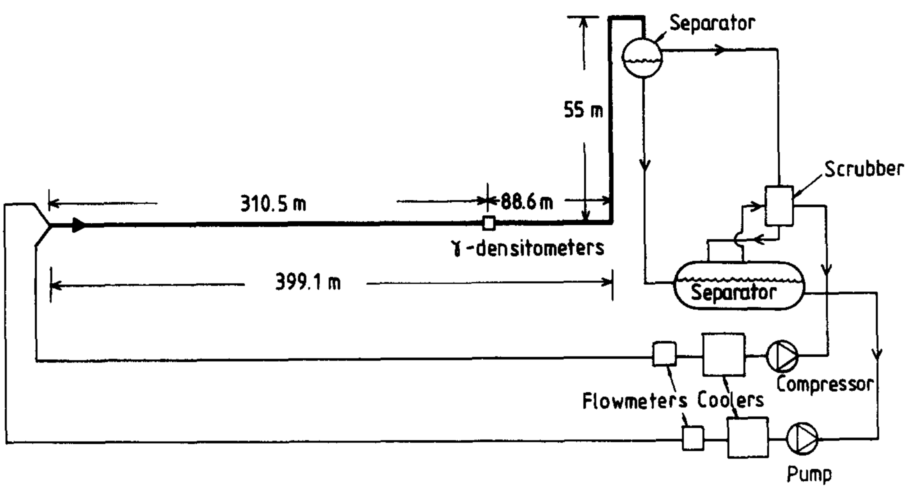

Figure 8. The SINTEF Two-phase Flow Lab.

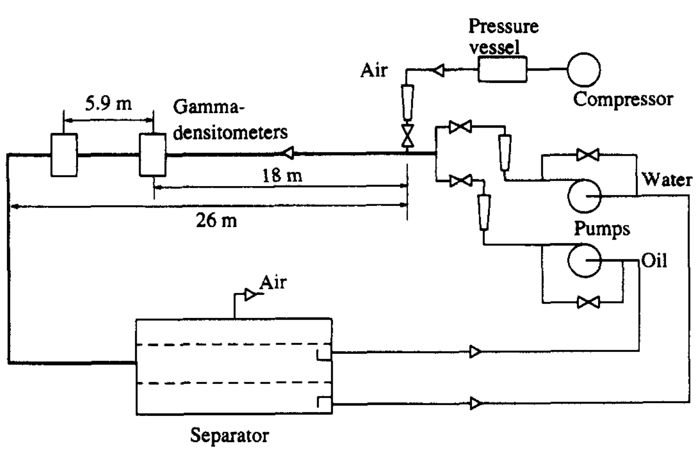

Figure 9. The IFE three-phase loop.

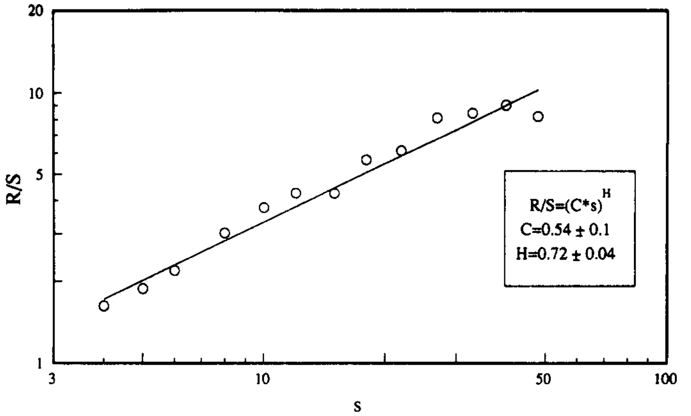

Figure 10. R/S as a function of s (○). The line represents a fit to $R/S = (Cs)^H$. (Data from the SINTEF loop with $U_m = 9.98$ m/s.)

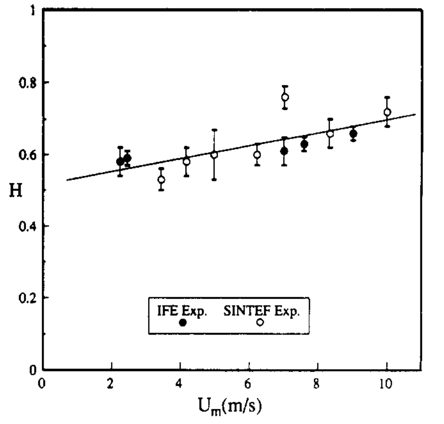

Figure 11. Hurst exponent (H) vs total superficial velocity (U_m).

measuring the time for the slug to travel between the two detectors, and the duration of each individual slug. Typically 30–50 slugs were recorded during each experiment.

The IFE three-phase loop has a pipe test section of length 26 m, with i.d. $\doteq$ 32 mm. The fluids applied were air, water and white spirit (EXXOL D-80) with an oil–water volumetric ratio of 10. The end of the pipe is vented to the atmosphere. The measuring station was positioned 18 m from the mixing point. The slug lengths were obtained from the recordings from two γ-densitometers placed 5.9 m apart. The dwell time was 20 ms. The experiments were run for approx. 1 min before the recordings were started. Typically 75–250 slugs were recorded for each experiment.

4. EXPERIMENTAL RESULTS AND DISCUSSION

The R/S analysis for the distribution of slug lengths was performed by plotting R/S vs s in a log–log plot based on [1]–[5]. The resulting slope gives the Hurst exponent. Figure 10 shows a typical example of R/S vs s for an experiment with total superficial velocity $U_m = 9.98$ m/s; $H = 0.72 \pm 0.04$, and the constant term is 0.63 ± 0.07 in the relationship $R/S = (Cs)^H$. Records of the R/S analysis for 45 bar large-scale slug length data from the SINTEF Two-phase Flow Lab. and from the IFE oil–water–air loop are presented in figure 11 as Hurst exponents vs total superficial velocity (U_m). The applied total superficial velocities for the 12 quoted experiments are given in table 1 with the corresponding Hurst exponents.

Two interesting effects are observed: the flow is persistent and it appears to be increasingly more so at higher flow rates. This at first glance unexpected result may in fact have been anticipated because of the high liquid flow rates applied. At low gas flow rates, plug flow with short slug bubbles

Table 1. Summary of experimental data

Loop	ϕ (deg)	U_m (m/s)	C	H
IFE	0	2.24	0.68 ± 0.18	0.58 ± 0.04
IFE	0	2.45	0.69 ± 0.09	0.59 ± 0.02
IFE	0	7.00	0.57 ± 0.14	0.61 ± 0.04
IFE	0	7.58	0.60 ± 0.06	0.63 ± 0.02
IFE	0	9.00	0.51 ± 0.07	0.66 ± 0.02
SINTEF	0	3.435	0.84 ± 0.14	0.53 ± 0.03
SINTEF	0	4.165	0.74 ± 0.15	0.58 ± 0.04
SINTEF	0	4.97	0.64 ± 0.22	0.60 ± 0.07
SINTEF	0	6.22	0.65 ± 0.10	0.60 ± 0.03
SINTEF	0	7.02	0.47 ± 0.05	0.76 ± 0.03
SINTEF	0	8.33	0.62 ± 0.11	0.66 ± 0.04
SINTEF	0	9.98	0.54 ± 0.10	0.72 ± 0.04

The inclination (ϕ) is relative to the horizontal, U_m is the total superficial velocity, C is the constant in the term $R/S = (Cs)^H$ and H is the Hurst exponent.

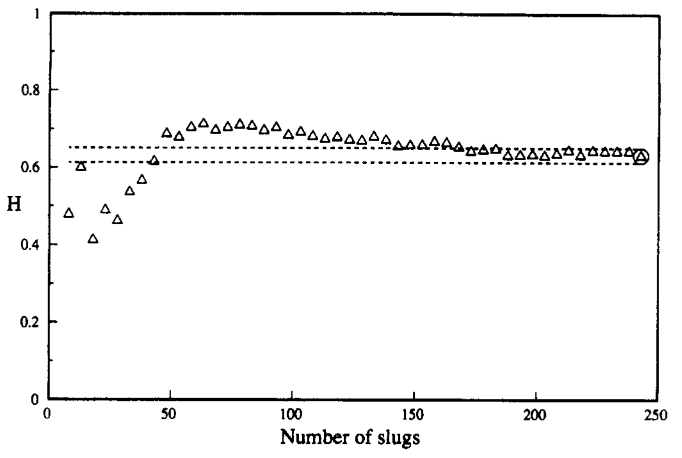

Figure 12. Hurst exponent (H) vs total number of slugs examined in one experiment from the IFE loop. The lines represent the standard deviation for the fit of R/S vs s for the maximum number of slugs in the experiment. The circle to the right identifies the value of H used in table 1.

of lengths between 2–10 dia were observed, and the flow is expected to develop quickly to a random distribution, as all bubble lengths are relatively short and constant. Increasing the gas flow rate causes longer bubbles, and each slug becomes increasingly dependent on the preceeding one, as well as on new slugs being generated in the liquid film under long slug bubbles. From the data, the degree of persistence expressed by the Hurst exponent might be expected to depend on the total superficial velocity (U_m), the inclination from the horizontal ($\Delta\phi$) and the development length or the mixing to measuring station distance (X_d^*):

$$H = H(U_m, \Delta\phi, X_d^*), \quad [18]$$

where

$$U_m = U_{sL} + U_{sG};$$

U_{sG} and U_{sL} are the superficial gas and liquid velocities, respectively.

Based on our data, a simple linear relationship is indicated:

$$H = 0.02U_m + 0.5. \quad [19]$$

This provides a quantitative measure of the degree to which any slug flow is developed or stable.

Objections regarding the number of events (slugs) being too small may be raised, as well as the downstream position of the measuring station, as a convenient measure of the past time history. To investigate the effect of the number of slugs recorded we have used data from the IFE loop. The Hurst exponent is shown to be relatively independent on the number of slugs analyzed, see figure 12.

The effect on persistence by different upstream flow development lengths should ideally be investigated in the large-scale facility by moving the measuring station. This, however, is expensive, and will have to be carried out in the future. An indication that the development length might indeed be sufficient, is provided by the agreement in results from the IFE loop with a development length on only 26 m (or 800D).

In order to predict the slug length distribution in hydrodynamic slug flow at a fixed point along a pipeline, the following parameters must first be determined: the Hurst exponent [19], the average slug length ($\bar{L}_s$) and the standard deviation (σ) for the liquid slug lengths.

Figure 13 shows a plot of measured and predicted liquid slug lengths vs slug number for one particular experiment from the IFE loop ($U_m = 7.58$ m/s); $\sigma = 0.23$, $L_s = 0.87$ m and $H = 0.63$. A total of 243 slugs were recorded. The predicted slug length distribution has the same properties (only 40 of the 256 slugs produced are shown in the figure). In particular, the predicted and measured maximum and minimum slug lengths are almost identical in the interval shown, 1.32 and 0.42 m, respectively.

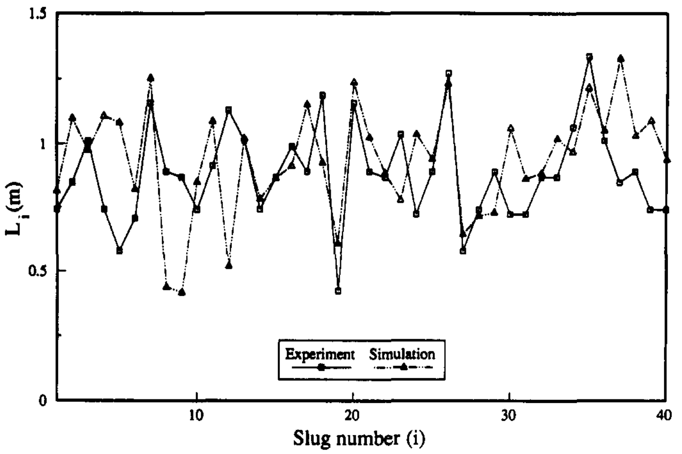

Figure 13. Measured and predicted liquid slug lengths as vs the slug number for the $U_m = 7.58$ m/s experiment in table 1 from the IFE loop; $\sigma = 0.23$, $\bar{L}_s = 0.87$ m and $H = 0.63$.

5. CONCLUSIONS

Liquid slug length distributions for horizontal flow obey fractal statistics.

An R/S analysis of available data shows that the data satisfies Hurst's law with a Hurst exponent in the range 0.53–0.76, linearly dependent on the total superficial velocity [19].

The method of successive random additions has been applied to generate slug length distributions when H, $\bar{L}_s$ and σ are known.

Predicted slug length distributions are very similar to observed ones. In particular, predicted and measured maximum and minimum liquid slug lengths are almost identical in a given experiment from the IFE loop; 1.32 and 0.42 m, respectively.

Fractal statistics thus provide a very promising method for quantitative analysis of slug flow data.

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